Exponentially temporally growing magnetic moment field of stimulated Brillouin scattering in plasma

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Stimulated Brillouin scattering generates a magnetic moment, due to the bending of the direction of motion of the plasma constituents, by the involved wave fields. A part of this moment is nonoscillating and grows exponentially in time, at twice the rate of growth of the local field. This important transfer of pump energy to the growing moment field, via the energy of the parametrically growing signal waves, has been evaluated in this paper. This field should ultimately control the growth character of the parametric instability, the synchrotron, and bremsstrahlung radiation losses, and the anisotropy of filamentation of the plasma. The field exists in both lateral and axial directions. The lateral field explains the anomalous diffusion of plasma.

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I. INTRODUCTION

Motion of plasma constituents in the presence of applied wave fields [and, equivalently, the wave induced displacement of electrically conducting fluids of magnetohydrodynamics (MHD)] generates a nonoscillating zero harmonic magnetic moment per unit volume. Formulation of several problems of wave induced moment field generation is possible for theoretical investigation. The most important of these problems are those in which this zero harmonic magnetization is found to grow exponentially in time, at twice the rate of the local fields. The nonlinearly excited parametric three-wave instability problems belong to this category.

This field was discovered in the interaction of strong waves of circular polarization, with crystals [1,2] in the 1970s, and subsequently with plasmas [3–7]. It was then called the inverse Faraday effect (IFE) field, because for circular polarization, it is essentially the inverse of the Faraday rotation effect. (N.B. Faraday rotation is the rotation of the plane of polarization of a plane polarized wave by an ambient magnetic field existing along its direction of propagation.) This nonoscillating, but temporally exponentially growing, IFE magnetization is generated instantaneously in stimulated Brillouin scattering (SBS) in plasmas. Since the loss through SBS is important in laser produced plasmas, this field must ultimately control the character of growth of the instability because its growth rate is highest (twice the growth rate of the excited wave fields). So, its prediction is an important achievement.

Reduction of laser light absorption due to the SBS effects is a concern for laser fusion applications. Any ambient magnetic field is not strong enough to affect greatly the propagation of a high frequency light wave, so the plasma may then be considered as unmagnetized. For the pump electromagnetic mode (photon), the possible decay modes are another electromagnetic (em) wave (photon), and either an electron acoustic wave (plasmon) of SBS, or an ion sound wave (phonons) of the SBS in plasma.

Phonons, created in the SBS process, can overcome their loss by decay and stimulate the light emission process, which excites the temporally growing magnetization. Since the em wave can propagate at any frequency above the plasma frequency, this resonant decay of an incident photon into a scattered photon plus an ion-acoustic phonon is possible throughout the region where the density is below the critical density. It gives rise to instability in the underdense region of an expanding plasma. Energy of the incoming mode is thus transferred into an excited em wave which may be backwards (or sideways) propagating light and thus scattered from the target instead of absorption. For an effective absorption of the incident laser light, the SBS should not be allowed to grow to a high level. And, since energy of the scattered photon and or the ion-acoustic phonon is proportional to the related wave frequency, most of the energy in SBS goes into the scattered photon because it is at a much higher frequency than the phonon. This highly amplified scattered light wave is the cause of the parametric instability. The temporally exponentially growing nonoscillating magnetization exists in both axial and lateral directions, determines the features of destabilization, and is responsible for strong synchrotron radiation and bremsstrahlung radiation from the region of the SBS. It augments the field of instability from other sources, including those of some nongrowing IFE fields [1,2,3,7].

The physical basis of SBS and its effect on inertial confinement fusion has been discussed in Sec. II. The basic field equations and assumptions are considered in Sec. III. Section IV contains the equations for the parametrically excited fields, and their solutions in the linearized approximation. These reduce to a set of Klein-Gordon wave equations, which have been discussed in Sec. V. Section VI is concerned with the evaluation of the magnetic moment field of the SBS in plasma. Numerically, this magnetization is estimated in Sec. VII in some cases. Some conclusions are drawn on the basis of our investigations in Sec. VIII.

II. PHYSICAL BASIS OF SBS IN LASER PRODUCED PLASMA

In laser produced plasma stimulated scattering effects reduce laser light absorption which greatly affects the target implosion in inertial confinement fusion (ICF) experiments. This scattering process involves the decay of an incident (pump) em wave into a scattered em wave and an acoustic wave (plasmon) which is electron acoustic for stimulated Raman scattering (SRS) and an ion-acoustic wave (phonon) for stimulated Brillouin scattering (SBS) processes.

In the SBS process the pump em wave couples with the scattered em wave and the ion-acoustic wave following some frequency and wave number matching conditions, so that the total energy and momentum of the system remain conserved. This coupling effectively transfers energy from the pump wave to the two daughter waves which in turn parametrically amplifies the daughter waves, leading ultimately to an instability which destabilizes the plasma very quickly. This is a drawback of ICF experiments.

Since an em wave can propagate at any frequency above the plasma frequency, and the frequency of ion-acoustic waves has no cutoff value, the stimulated Brillouin scattering is possible throughout the underdense region of a long scale length plasma, produced by a solid target irradiated by a high intensity long pulse laser.

As the generation of SRS and SBS instability depends on the scale length of the plasma intensity and pulse length of the laser beam, by controlling these parameters these two processes can be avoided. The SBS is not only injurious, it also has some good impact on inertial confinement fusion [8,9]. The noncollisional damping of electron plasma waves in the SRS process generates fast electrons, which corresponds to nonuniform target implosion. When SRS and SBS coexist in plasma, the SBS can suppress this fast electron generation of the SRS mechanism. For understanding the different linear and nonlinear aspects of SBS, including this suppression mechanism, the role of the nonlinearly parametrically amplified IFE type magnetization has not yet been investigated. In Sec. VI we have opened a discussion in this regard.

III. BASIC FIELD EQUATIONS AND ASSUMPTIONS

Since ambient magnetic field is not strong enough to affect greatly the propagation of high frequency light waves, its effect is neglected here. Moreover, the plasma is highly collisional; since this collision is randomized, its average effect is neglected. So, we consider the following system of equations:

$$mn_e\left[\frac{\partial \mathbf{u}_e}{\partial t}+(\mathbf{u}_e\cdot\nabla)\mathbf{u}_e\right]=-k_B T_e \nabla n_e - en_e\mathbf{E} - \frac{en_e}{c}(\mathbf{u}_e\times\mathbf{H}) , \quad (3.1)$$

$$Mn_i\left[\frac{\partial \mathbf{u}_i}{\partial t}+(\mathbf{u}_i\cdot\nabla)\mathbf{u}_i\right]=-k_B T_i \nabla n_i + en_i\mathbf{E} + \frac{en_i}{c}(\mathbf{u}_i\times\mathbf{H}) , \quad (3.2)$$

$$\frac{\partial n_e}{\partial t}+\operatorname{div}(n_e\mathbf{u}_e)=0 , \quad (3.3)$$

$$\frac{\partial n_i}{\partial t}+\operatorname{div}(n_i\mathbf{u}_i)=0 , \quad (3.4)$$

$$\nabla\times\mathbf{E}=-\frac{1}{c}\frac{\partial \mathbf{H}}{\partial t} , \quad (3.5)$$

$$\nabla\times\mathbf{H}=\frac{1}{c}\frac{\partial \mathbf{E}}{\partial t}+\frac{4\pi e}{c}(N_i\mathbf{u}_i-N_e\mathbf{u}_e) , \quad (3.6)$$

$$\nabla\cdot\mathbf{E}=4\pi e(n_i-n_e) , \quad (3.7)$$

$$\nabla\cdot\mathbf{H}=0 , \quad (3.8)$$

where $\nabla\equiv(0,0,\partial/\partial z)$; $n_e,\mathbf{v}_e$ are the wave induced number density and velocity of electrons, n_i and $\mathbf{v}_i$ are those of ions, and $\mathbf{E}$ and $\mathbf{H}$ are the net electric and magnetic fields of the system; OZ is the common direction of propagation of all three waves. The compressibility of the plasma generates the ion-acoustic wave in SBS processes. Here we consider only the Brillouin backscattering or forward scattering; the case of sideband scattering is currently excluded to avoid mathematical complexity. The subscripts 1 and 2 stand for the incident and scattered light waves, and the subscript 3 represents the ion-acoustic wave. So, this plasma is a mixture of fully ionized, collision free, unmagnetized, and compressible fluids of negative and positive charges. Initially, this plasma is quasistatic and quasineutral with unperturbed density n_0, for both electrons and ions. In the three-wave SBS program, the electric field, the magnetic field, and the density and velocity of the electron and ion components, in the first order perturbation [4], are

$$\begin{aligned} n_e&=n_0+n_3 ,\\ n_i&=n_0+N_3 ,\\ \boldsymbol{v}_e&=\boldsymbol{v}_1+\boldsymbol{v}_2+v_3\hat{\mathbf{z}} ,\\ \mathbf{v}_i&=\mathbf{V}_1+\mathbf{V}_2+V_3\hat{\mathbf{z}} ,\\ \mathbf{E}&=\mathbf{E}_1+\mathbf{E}_2+E_3\hat{\mathbf{z}} ,\\ \mathbf{H}&=\mathbf{H}_1+\mathbf{H}_2 . \end{aligned} \quad (3.9)$$

Here $\boldsymbol{v}$ and $\mathbf{V}$ are the first order perturbations in the velocities of electrons and ions and n and N are the first order perturbations in the corresponding number densities. Velocities of the electrons and ions, induced by the third wave, have only longitudinal components. The electric fields of the three waves, therefore, are

$$\begin{aligned} \mathbf{E}_1&=(a_1\cos\theta_1, b_1\sin\theta_1, 0) ,\\ \mathbf{E}_2&=(a_2\cos\theta_2, b_2\sin\theta_2, 0)\\ \mathbf{E}_3&=(0,0,a_3\cos\theta_3) , \end{aligned} \quad (3.10)$$

where $\theta_i = k_i z - \omega_i t$, $i = 1,2,3$, and k_i and ω_i are the wave number and frequency of the respective waves. The resonance conditions and the dispersion relations for the three waves, approximately, are [10]

$$\omega_1 = \omega_2 + \omega_3 \ , \quad k_1 = k_2 + k_3 \ , \tag{3.11}$$

$$k_1^2 c^2 \approx \omega_1^2 \ , \quad k_2^2 c^2 \approx \omega_2^2 \ , \quad k_3^2 c_s^2 \approx \omega_3^2 \ , \tag{3.12}$$

where c is the velocity of light in vacuum, c_s $(=\sqrt{k_B T_e / M})$ is the phase velocity of the sound wave in plasma, k_B is the Boltzmann constant, and the electron temperature T_e is generally large compared to the ion temperature T_i. For perfect matching of the frequencies, the wave number mismatch is negligibly small, so both the conditions of (3.11) are valid, and for the phases give the phase matching condition

$$\theta_1 = \theta_2 + \theta_3 \ . \tag{3.13}$$

IV. PARAMETRIC EQUATIONS AND THEIR LINEARIZED FIELD SOLUTIONS

Using (3.9) in (3.1)–(3.8), we obtain the following parametric equations for the incident light wave and the two decay waves, describing their evolution in the presence of the pump wave:

$$\frac{\partial \mathbf{v}_1}{\partial t} + \frac{e}{m} \mathbf{E}_1 = \mathbf{0} \ , \tag{4.1}$$

$$\frac{\partial \mathbf{V}_1}{\partial t} - \frac{e}{M} \mathbf{E}_1 = \mathbf{0} \ , \tag{4.2}$$

$$\frac{\partial \mathbf{v}_2}{\partial t} + \frac{e}{m} \mathbf{E}_2 = -v_3 \frac{\partial \mathbf{v}_1}{\partial z} - \frac{e}{mc} (v_3 \hat{\mathbf{z}} \times \mathbf{H}_1) \ , \tag{4.3}$$

$$\frac{\partial \mathbf{V}_2}{\partial t} - \frac{e}{M} \mathbf{E}_2 = -V_3 \frac{\partial \mathbf{V}_1}{\partial z} + \frac{e}{Mc} (V_3 \hat{\mathbf{z}} \times \mathbf{H}_1) \ , \tag{4.4}$$

$$\frac{\partial v_3}{\partial t} + \frac{e}{m} E_3 + \frac{c_e^2}{n_0} \frac{\partial n_3}{\partial z} = -\frac{e}{mc} [(\mathbf{v}_1 \times \mathbf{H}_2) + (\mathbf{v}_2 \times \mathbf{H}_1)] \ , \tag{4.5}$$

$$\frac{\partial \mathbf{V}_3}{\partial t} - \frac{e}{M} E_3 + \frac{c_i^2}{n_0} \frac{\partial N_3}{\partial z} = \frac{e}{Mc} [(\mathbf{V}_1 \times \mathbf{H}_2) + (\mathbf{V}_2 \times \mathbf{H}_1)] \ , \tag{4.6}$$

$$\frac{\partial n_3}{\partial t} + n_0 \frac{\partial v_3}{\partial z} = 0 \ , \tag{4.7}$$

$$\frac{\partial N_3}{\partial t} + n_0 \frac{\partial V_3}{\partial z} = 0 \ , \tag{4.8}$$

$$\frac{\partial E_3}{\partial z} = 4\pi e (N_3 - n_3) \ . \tag{4.9}$$

The homogeneous equations (4.1) and (4.2) describe the incident pump light wave. The inhomogeneous equations for the scattered light wave are (4.3) and (4.4). Their right hand sides are the sources of its excitation; since the sources are exclusively from the parametric coupling between the incident light wave and the ion-acoustic wave, when subjected to the phase matching condition (3.13), only the relevant terms are retained. Similarly, for the parametrically excited ion-acoustic wave, the right hand sides of the inhomogeneous equations (4.5)–(4.9) contain only the terms of parametric coupling between the incident and scattered light waves. At the initial stage of the Brillouin instability, the decay waves are weak enough and do not have an appreciable influence on the pump wave. So, the coupling terms on the right hand sides of Eq. (4.1) and (4.2) are neglected.

Using (3.10) in the linearized approximation of (4.1)–(4.8), we obtain the following linearized field solutions:

$$\mathbf{v}_1 = \frac{e}{m\omega_1} (a_1 \sin\theta_1, -b_1 \cos\theta_1, 0) \ , \tag{4.10}$$

$$\mathbf{V}_1 = -\frac{e}{M\omega_1} (a_1 \sin\theta_1, -b_1 \cos\theta_1, 0) \ , \tag{4.11}$$

$$\mathbf{v}_2 = \frac{e}{m\omega_2} (a_2 \sin\theta_2, -b_2 \cos\theta_2, 0) \ , \tag{4.12}$$

$$\mathbf{V}_2 = -\frac{e}{M\omega_2} (a_2 \sin\theta_2, -b_2 \cos\theta_2, 0) \ , \tag{4.13}$$

$$v_3 = -\frac{e\omega_3}{k_3^2 m (c_e^2 - c_s^2)} a_3 \sin\theta_3 \ , \tag{4.14}$$

$$V_3 = \frac{e\omega_3}{k_3^2 M (c_i^2 - c_s^2)} a_3 \sin\theta_3 \ , \tag{4.15}$$

$$n_3 = -\frac{e n_0}{k_3 m (c_e^2 - c_s^2)} a_3 \sin\theta_3 \ , \tag{4.16}$$

$$N_3 = \frac{e n_0}{k_3 M (c_i^2 - c_s^2)} a_3 \sin\theta_3 \ . \tag{4.17}$$

The linearized Maxwell equations give

$$\mathbf{H}_1 = \frac{k_1 c}{\omega_1} (-b_1 \sin\theta_1, a_1 \cos\theta_1, 0) \ , \tag{4.18}$$

$$\mathbf{H}_2 = \frac{k_2 c}{\omega_2} (-b_2 \sin\theta_2, a_2 \cos\theta_2, 0) \ . \tag{4.19}$$

V. THE KLEIN-GORDON WAVE EQUATIONS FOR PARAMETRICALLY EVOLVED FIELDS

The SBS of an Alfven wave (which is of frequency less than the ion gyration frequencies), and a sound wave, by a pump Alfven wave, in a perfect MHD fluid, have been much studied [11]. In laser produced plasmas, the SBS can occur throughout the underdense region where density variation affects the plasma dielectric constant. Consequently, the incident electrical energy is transferred to the scattered electric and acoustic fields; the maximum amount of energy is drawn by scattered light wave, because it is of very high frequency, and frequency of the ion-acoustic wave is small.

Equations (4.1)–(4.8), for parametric evolution of fields, yield the following Klein-Gordon (KG) wave equations:

$$L\mathbf{E}_1=0\ ,\quad L\mathbf{H}_1=0\ , \tag{5.1}$$

$$\begin{aligned} L\mathbf{E}_2=&4\pi e n_0\left[V_3\frac{\partial \mathbf{V}_1}{\partial z}\right]-4\pi e n_0\left[v_3\frac{\partial \mathbf{v}_1}{\partial z}\right] \\ &-\frac{\omega_{b_i}^2}{c}(V_3\hat{\mathbf{z}}\times\mathbf{H}_1)-\frac{\omega_{p_e}^2}{c}(v_3\hat{\mathbf{z}}\times\mathbf{H}_1) \\ &-4\pi e\frac{\partial}{\partial t}(N_3\mathbf{V}_1)+4\pi e\frac{\partial}{\partial t}(n_3\mathbf{v}_1)\ , \end{aligned} \tag{5.2}$$

$$\begin{aligned} L\dot{\mathbf{H}}_2=-c\nabla\times\Bigg[&4\pi e n_0\left[V_3\frac{\partial \mathbf{V}_1}{\partial z}\right]-4\pi e n_0\left[v_3\frac{\partial \mathbf{v}_1}{\partial z}\right] \\ &-\frac{\omega_{p_i}^2}{c}(V_3\hat{\mathbf{z}}\times\mathbf{H}_1)-\frac{\omega_{p_e}^2}{c}(v_3\hat{\mathbf{z}}\times\mathbf{H}_1) \\ &-4\pi e\frac{\partial}{\partial t}(N_3\mathbf{V}_1)+4\pi e\frac{\partial}{\partial t}(n_3\mathbf{v}_1)\Bigg]\ , \end{aligned} \tag{5.3}$$

$$\frac{\partial E_3}{\partial t}+4\pi e n_0(V_3-v_3)=0\ , \tag{5.4}$$

$$\begin{aligned} L_1^2 v_3=&-\frac{e}{mc}L_i\frac{\partial}{\partial t}[(\mathbf{v}_1\times\mathbf{H}_2)+(\mathbf{v}_2\times\mathbf{H}_1)]_{\parallel} \\ &+\frac{e}{Mc}\omega_{p_e}^2\frac{\partial}{\partial t}[(\mathbf{V}_1\times\mathbf{H}_2)+(\mathbf{V}_2\times\mathbf{H}_1)]_{\parallel}\ , \end{aligned} \tag{5.5}$$

$$\begin{aligned} L_1^2 V_3=&\frac{e}{Mc}L_e\frac{\partial}{\partial t}[(\mathbf{V}_1\times\mathbf{H}_2)+(\mathbf{V}_2\times\mathbf{H}_1)]_{\parallel} \\ &-\frac{e}{mc}\omega_{p_i}^2\frac{\partial}{\partial t}[(\mathbf{v}_1\times\mathbf{H}_2)+(\mathbf{v}_2\times\mathbf{H}_1)]_{\parallel}\ , \end{aligned} \tag{5.6}$$

$$\begin{aligned} L_1^2 n_3=&\frac{e n_0}{mc}L_i\frac{\partial}{\partial z}[(\mathbf{v}_1\times\mathbf{H}_2)+(\mathbf{v}_2\times H_1)]_{\parallel} \\ &-\frac{e n_0}{Mc}\omega_{p_e}^2\frac{\partial}{\partial z}[(\mathbf{V}_1\times H_2)+(\mathbf{V}_2\times H_1)]_{\parallel}\ , \end{aligned} \tag{5.7}$$

$$\begin{aligned} L_1^2 N_3=&-\frac{e n_0}{Mc}L_e\frac{\partial}{\partial z}[(\mathbf{V}_1\times\mathbf{H}_2)+(\mathbf{V}_2\times\mathbf{H}_1)]_{\parallel} \\ &+\frac{e n_0}{mc}\omega_{p_i}^2\frac{\partial}{\partial z}[(\mathbf{v}_1\times\mathbf{H}_2)+(\mathbf{v}_2\times\mathbf{H}_1)]_{\parallel} \end{aligned} \tag{5.8}$$

for the three waves, where

$$\begin{aligned} &L\equiv\frac{\partial^2}{\partial t^2}-c^2\frac{\partial^2}{\partial z^2}+\omega_{p_e}^2+\omega_{p_i}^2\ ,\quad L_1\equiv L_e L_i-\omega_{p_e}^2\omega_{p_i}^2\ , \\ &L_e\equiv\frac{\partial^2}{\partial t^2}-c_e^2\frac{\partial^2}{\partial z^2}+\omega_{p_e}^2\ ,\quad L_i\equiv\frac{\partial^2}{\partial t^2}-c_i^2\frac{\partial^2}{\partial z^2}+\omega_{p_i}^2\ . \end{aligned} \tag{5.9}$$

The KG equations of (5.1), for the pump wave, are homogeneous, because its parametric or any other nonlinear type of evolution is not important in the effect of this study. However, for the important parametric evolution of the other two fields, the relevant inhomogeneities of the KG equations (5.2)–(5.8) are the sources of parametric evolution of the idler fields. Now, substituting the linearized solutions (4.10)–(4.19) in the right hand sides of (5.2)–(5.8), and using the phase matching condition (3.13), we obtain the relations

$$-4\pi e n_0\left[v_3\frac{\partial \mathbf{v}_1}{\partial z}\right]-\frac{\omega_{p_e}^2}{c}(v_3\hat{\mathbf{z}}\times\mathbf{H}_1)=0\ , \tag{5.10}$$

$$4\pi e n_0\left[V_3\frac{\partial \mathbf{V}_1}{\partial z}\right]-\frac{\omega_{p_i}^2}{c}(V_3\hat{\mathbf{z}}\times\mathbf{H}_1)=0\ , \tag{5.11}$$

and the more explicit inhomogeneous wave equations,

$$\begin{aligned} L\mathbf{E}_2=&\frac{e}{2k_3\omega_1}\left[\frac{\omega_{p_i}^2}{M(c_i^2-c_s^2)}-\frac{\omega_{p_e}^2}{m(c_e^2-c_s^2)}\right] \\ &\times\frac{\partial}{\partial t}(a_3a_1\cos\theta_2,a_3b_1\sin\theta_2,0)\ , \end{aligned} \tag{5.12}$$

$$\begin{aligned} L\mathbf{H}_2=&\frac{ce}{2K_3\omega_1}\left[\frac{\omega_{p_i}^2}{M(c_i^2-c_s^2)}-\frac{\omega_{p_e}^2}{m(c_e^2-c_s^2)}\right] \\ &\times\frac{\partial}{\partial z}(a_3b_1\sin\theta_2,-a_3a_1\cos\theta_2,0)\ , \end{aligned} \tag{5.13}$$

$$\begin{aligned} L_1^2 v_3=&\frac{e^2k_3}{2\omega_1\omega_2}\left[\frac{L_i}{m^2}+\frac{\omega_{p_e}^2}{M^2}\right] \\ &\times\frac{\partial}{\partial z}[(a_1a_2+b_1b_2)\sin\theta_2]\ , \end{aligned} \tag{5.14}$$

$$\begin{aligned} L_1^2 n_3=&-\frac{e^2n_0k_3}{2\omega_1\omega_2}\left[\frac{L_i}{m^2}+\frac{\omega_{p_e}^2}{M^2}\right] \\ &\times\frac{\partial}{\partial z}[(a_1a_2+b_1b_2)\sin\theta_2]\ , \end{aligned} \tag{5.15}$$

$$\begin{aligned} L_1^2 V_3=&\frac{e^2k_3}{2\omega_1\omega_2}\left[\frac{L_e}{M^2}+\frac{\omega_{p_i}^2}{m^2}\right] \\ &\times\frac{\partial}{\partial t}[(a_1a_2+b_1b_2)\sin\theta_2]\ , \end{aligned} \tag{5.16}$$

$$\begin{aligned} L_1^2 N_3=&\frac{e^2n_0k_3}{2\omega_1\omega_2}\left[\frac{L_e}{M^2}+\frac{\omega_{p_i}^2}{m^2}\right] \\ &\times\frac{\partial}{\partial z}[(a_1a_2+b_1b_2)\sin\theta_2]\ . \end{aligned} \tag{5.17}$$

For parametric amplification of the scattered light wave and ion-acoustic wave, we assume that the amplitudes of these decay waves are slowly varying functions of time only. Their first order time derivatives will be retained. The linearized dispersion relations are used on the left hand sides of these equations and the phase matching condition (3.13) on their right hand sides. Then, equating the coefficients of sine and cosine functions of $(KZ-\omega t)$ from both sides, we obtain

$$\dot{a}_2=\frac{e}{4k_3\omega_1}\left[\frac{\omega_{p_i}^2}{M(c_i^2-c_s^2)}-\frac{\omega_{p_e}^2}{m(c_e^2-c_s^2)}\right]a_3a_1\ , \tag{5.18}$$

$$\dot{b}_2=\frac{e}{4k_3\omega_1}\left[\frac{\omega_{p_i}^2}{M(c_i^2-c_s^2)}-\frac{\omega_{p_e}^2}{m(c_e^2-c_s^2)}\right]a_3b_1\ , \tag{5.19}$$

$$\dot{a}_3=-\frac{ek_3\omega_3 m^2(c_e^2-c_s^2)}{4\omega_1\omega_2 M c_s^2(k_3^2c_i^2+\omega_{p_1}^2)}\times\left[\frac{\omega_{p_e}^2}{M}+\frac{k_3^2c_i^2-\omega_3^2+\omega_{p_i}^2}{m}\right](a_1a_2+b_1b_2) \quad (5.20)$$

for parametric evolution of the idler fields in time. These give

$$a_1\ddot{a}_2+b_1\ddot{b}_2=-\frac{e^2m^2c_e^2\omega_3}{16\omega_1^2\omega_2 M c_s^2(k_3^2c_i^2+\omega_{p_i}^2)}\times\left[\frac{\omega_{p_i}^2}{M(c_i^2-c_s^2)}-\frac{\omega_{p_e}^2}{m(c_e^2-c_s^2)}\right]\times\left[\frac{\omega_{p_e}^2}{M^2}+\frac{k_3^2c_i^2-\omega_3^2+\omega_{p_i}^2}{m^2}\right]\times(a_1a_2+b_1b_2)(a_1^2+b_1^2)\,. \quad (5.21)$$

Since, for parametric evolution problems, a_2 and b_2 are slowly varying functions of time only, we assume $a_2=a_{20}e^{\gamma t}$ and $b_2=b_{20}e^{\gamma t}$, and obtain

$$\gamma^2=-\frac{e^2m^2c_e^2\omega_3}{16Mc_s^2\omega_1^2\omega_2(k_3^2c_i^2+\omega_{p_i}^2)}\times\left[\frac{\omega_{p_i}^2}{M(c_i^2-c_s^2)}-\frac{\omega_{p_e}^2}{m(c_e^2-c_s^2)}\right]\times\left[\frac{\omega_{p_e}^2}{M^2}+\frac{k_3^2c_i^2-\omega_3^2+\omega_{p_i}^2}{m^2}\right](a_1^2+b_1^2). \quad (5.22)$$

If we neglect the effect of ion temperature ($T_i \ll T_e$), then $c_i^2 \ll c_e^2$, which implies

$$\frac{\omega_{p_i}^2}{M(c_i^2-c_s^2)}\ll\frac{\omega_{p_e}^2}{m(c_e^2-c_s^2)}\,. \quad (5.23)$$

And since, for the ion-acoustic wave, $\omega_3 \ll \omega_p$, we obtain

$$\frac{\omega_{p_e}^2}{M^2}+\frac{k_3^2c_i^2-\omega_3^2+\omega_{p_i}^2}{m^2}\approx\frac{k_3^2c_i^2+\omega_{p_i}^2}{m^2}\,, \quad (5.24)$$

$$\gamma^2=\frac{e^2\omega_{p_e}^2\omega_3}{16\omega_1^2\omega_2 mM}\frac{c_e^2}{c_s^2(c_e^2-c_s^2)}(a_1^2+b_1^2)\,. \quad (5.25)$$

And because $c_e^2 > c_s^2$, a parametric instability develops for real values of γ, if ω_2 and ω_3 are of the same sign. In other words, parametric instability occurs if the scattered light wave and the ion-acoustic wave propagate in the same direction. If they propagate in the forward direction, parametric instability may enhance the filamentation instability, otherwise there will be interference of light of incident and scattering fields.

VI. MAGNETIC MOMENT FIELD FROM SBS IN PLASMA

In a laser produced electron plasma, the moment field (the IFE magnetization) was shown to be about 10^5 G by Steiger and Woods [3], the effect of ion motion in it was included by Chain [4].

In the SBS process, energy is effectively transferred from the pump light wave to two daughter waves (one light wave and an ion-acoustic wave), highly amplifying these, keeping the momentum and energy conserved. This amplification of waves distorts the motion of the charged particles, generating, thereby, the magnetic moment field which has a nonoscillating, but temporally growing in time, part.

The magnetic dipole moment from bending of motion of electrons and ions by the scattered light wave, and ion-acoustic wave, can be written as

$$\boldsymbol{\mu}=\boldsymbol{\mu}_e+\boldsymbol{\mu}_i\,, \quad (6.1a)$$

where

$$\boldsymbol{\mu}_e=\frac{1}{2c}[(\mathbf{r}_2\times\mathbf{j}_2)+(\mathbf{r}_2\times\mathbf{j}_3)+(\mathbf{r}_3\times\mathbf{j}_2)]\,,$$
$$\boldsymbol{\mu}_i=\frac{1}{2c}[(\mathbf{R}_2\times\mathbf{J}_2)+(\mathbf{R}_2\times\mathbf{J}_3)+(\mathbf{R}_3\times\mathbf{J}_2)]\,, \quad (6.1b)$$

where $\mathbf{r}_2$ and $\mathbf{R}_2$ are the displacements of electrons and ions, and $\mathbf{j}_2,\mathbf{J}_2$ are the corresponding currents induced by the scattered light wave , so that

$$\mathbf{j}_2=-e\mathbf{v}_2 \text{ and } \mathbf{J}_2=e\mathbf{V}_2\,. \quad (6.2)$$

Similarly $\mathbf{r}_3,\mathbf{R}_3,\mathbf{j}_3,\mathbf{J}_3$ are the displacements and currents of electrons and ions, induced by the ion-acoustic wave. Integrating the linearized solutions of $\mathbf{v}_2,\mathbf{V}_2$ and v_3,V_3, from (4.12)–(4.15) with respect to time, we obtain the displacements

$$\mathbf{r}_2=\frac{e\exp(\gamma t)}{m\omega_2(\gamma^2+\omega_2^2)}(a_{20}(\omega_2\cos\theta_2+\gamma\sin\theta_2),-b_{20}(\gamma\cos\theta_2-\omega_2\sin\theta_2),0)\,, \quad (6.3)$$

$$\mathbf{R}_2=-\frac{e\exp(\gamma t)}{M\omega_2(\gamma^2+\omega_2^2)}(a_{20}(\omega_2\cos\theta_2+\gamma\sin\theta_2),-b_{20}(\gamma\cos\theta_2-\omega_2\sin\theta_2),0)\,, \quad (6.4)$$

$$\mathbf{r}_3=-\frac{e\omega_3\exp(\gamma t)a_{30}}{k_3^2m(c_e^2-c_s^2)(\gamma^2+\omega_3^2)}(\omega_3\cos\theta_3+\gamma\sin\theta_3)\hat{\mathbf{z}}\,, \quad (6.5)$$

$$\mathbf{R}_3=\frac{e\omega_3\exp(\gamma t)a_{30}}{k_3^2M(c_i^2-c_s^2)(\gamma^2+\omega_3^2)}(\omega_3\cos\theta_3+\gamma\sin\theta_3)\hat{\mathbf{z}}\,. \quad (6.6)$$

Hence, with respect to the common direction of wave propagation, the $(\mathbf{r}_2\times\mathbf{j}_2)$ and $(\mathbf{R}_2\times\mathbf{J}_2)$ terms in $\boldsymbol{\mu}_e$ and $\boldsymbol{\mu}_i$ are longitudinal whereas $(\mathbf{r}_2\times\mathbf{j}_3)$, $(\mathbf{r}_3\times\mathbf{j}_2)$ and $(\mathbf{R}_2\times\mathbf{J}_3)$, $(\mathbf{R}_3\times\mathbf{J}_2)$ are transverse. Thus the magnetic moment field generated from SBS in laser produced plasma has both longitudinal and transverse components. Since each component of a displacement, obtained from (6.3)–(6.6), is directly proportional to $\exp(\gamma t)$, the velocity components have the same exponential dependence with time. The relevant part of the magnetic moment of (6.1a) and (6.1b) is proportional to $\exp(2\gamma t)$. The axial and lateral components of this part have been evaluated. These are

$$H_{\parallel}=\frac{e^3c_s^2\omega_{p_e}^2\exp(2\gamma t)}{32mc\omega_1^2\omega_2\omega_3^2\gamma^2(\gamma^2+\omega_2^2)}\times\left[\frac{\omega_{p_e}^2}{m(c_e^2-c_s^2)}-\frac{\omega_{p_i}^2}{M(c_i^2-c_s^2)}\right]a_{30}^2a_1^2\ , \qquad (6.7)$$

$$H_{\perp}=-\frac{e^2c_s^3\exp(2\gamma t)}{16c\gamma\omega_1\omega_2\omega_3^2\sqrt{\gamma^2+\omega_3^2}}\left[\frac{\omega_{p_e}^2}{m(c_e^2-c_s^2)}-\frac{\omega_{p_i}^2}{M(c_i^2-c_s^2)}\right]\times\left[1-\frac{2(\gamma^2+\omega_2\omega_3)}{\gamma^2+\omega_2^2}\right]a_{30}^2a_1\ . \qquad (6.8)$$

The axial field in (6.7) is generated by the self-interaction of the parametrically evolved scattered light wave whereas the field (6.8) in the lateral direction is generated by the mutual interaction of the parametrically evolved scattered light wave and ion-acoustic wave. The zero harmonic part of the lateral field exists only for circularly polarized signal em waves. The growth rate of both the moment fields $H_{\parallel}$ and $H_{\perp}$ is twice the growth rate of parametric instability. Hence, effectively, it controls the growth character of the parametric instability during the SBS process in laser plasma experiments. Consequences are bunching of plasma and energy evaporation by strong synchrotron radiation and bremsstrahlung radiation, acceleration of protons and electrons along the direction of wave propagation, and an anisotropy for the fast particle distributions for generation of other Alfven waves by accelerating particles [12]. Thus, in laser plasma experiments, the fast electrons generated from noncollisional damping of acoustic waves in stimulated scattering effects will be trapped quickly on the lines of forces of the fast generating IFE field, reducing the nonuniformity of target implosion in inertial confinement fusion.

VII. NUMERICAL ESTIMATION

Consider an underdense plasma ($n\sim10^{20}$ cm^{-3}), produced by a pump laser field ($\lambda=1.06$ μm) at intensity $I\sim5\times10^{14}$ W/cm^2, electron temperature $k_BT_e\sim1$ KeV, and pump frequency $\omega_1\sim1.78\times10^{15}$ s^{-1}; then the electron and ion plasma frequencies are 5.64×10^{14} s^{-1} and 1.32×10^{13} s^{-1}, respectively. Let the frequency of the ion-acoustic wave be $\omega_3\sim5\times10^5$ s^{-1}. Since $\omega_2=\omega_1-\omega_3$, the frequency of the scattered light wave may be 10^{15} s^{-1}. Let the sound speed be $c_s=3.2\times10^7$ cm/sec. The thermal velocity of electrons is $c_e=1.34\times10^9$ cm/sec. The formula for $P_{\perp}$, the power flux of the em wave being $P_{\perp}=(c/4\pi)(a_1^2+b_1^2)$, setting $a_1^2+b_1^2=2.1\times10^{12}$ ergs/cm^3, the growth rate γ is found to be 10^9 s^{-1}. Then the time of growth of this instability is 1 ns. Now, assume that an ion-acoustic wave of amplitude 1.5 V/m propagates (which has been obtained by neglecting the electron inertia of the linearized equation of motion satisfied by the electrons in a compressible plasma in the presence of electrostatic fields), and the density perturbation is 10^{17} cm^{-3}. Then the fields 300 G and 800 G are obtained at $t=0$ in axial and lateral direction, respectively. These fields that exponentially increase with time destabilize the plasma.

VIII. CONCLUSIONS AND CONCLUDING REMARKS

Any change of direction of motion of charges, in the presence of some waves in a plasma, including waves subjected to a specified type of frequency matching (as in nonlinear parametric processes), gives rise to a field of magnetic moment per unit volume. It has a static part, which is exponentially temporally growing, but nonoscillating at the same time, in problems of temporal evolution of fields in nonlinear, parametric-wave–plasma interactions. This extension of the scope of the IFE field has thus enabled us to find the moment field of SBS in plasma. The decay waves and the incident light wave are coupled with each other in the underdense region, which causes the transfer of energy from the incident wave to the two decay waves. Then parametric amplification of the decay waves causes a parametric instability which enhances the filamentation instability for forward scattering and interference of light waves for backward scattering. This parametric instability also generates a temporally exponentially growing IFE type of magnetization in both axial and lateral directions. These, being the fastest growing fields, determine the character of this destabilization mechanism. In a collisionally damped plasma, the instability may occur when the pump power exceeds some threshold value. The bremsstrahlung radiation, though generally insignificant in the low density region, becomes pronounced due to the local plasma bunching caused by the increasing magnetic pressure of the growing field in low density plasmas. The increasing magnetic pressure also gives rise to strong synchrotron radiation for a short duration.

[1] P. S. Pershan, Phys. Rev. **130**, 919 (1963).
[2] P. S. Pershan, J. P. Vanderziel, and L. D. Malmstrom, Phys. Rev. **143**, 574 (1966).
[3] A. D. Steiger and C. H. Woods, Phys. Rev. A **5**, 1467 (1972).
[4] A. C. L. Chain, Phys. Fluids **24**, 369 (1981).
[5] B. Chakraborty, M. R. Khan, Susmita Sarkar, V. Krishan, and B. Bhattacharyya, Ann. Phys. (N.Y.) **201**, 1 (1990).
[6] Y. Pomeau and D. Quemada, C. R. Acad. Sci. Ser. B **264**, 517 (1967).
[7] Susmita Sarkar, B. Bera, Manoranjan Khan, and B. Chakraborty, Aust. J. Phys. **44**, 59 (1991).
[8] C. H. Aldrich, B. Bezzerides, D. F. DuBois, and M. A. Rose, Comments Plasma Phys. Controlled Fusion **10**, 1 (1986).
[9] W. Rozmus, R. P. Sharma, J. C. Samson, and W. Tighe, Phys. Fluids **30**, 2181 (1987).
[10] W. L. Kruer, *The Physics of Laser Plasma Interactions* (Addison-Wesley, Reading, MA, 1988), p. 88.
[11] R. Z. Sagdeev and A. A. Galeev, *Nonlinear Plasma Theory* (Benjamin, New York, 1969).
[12] I. N. Toptygin, *Cosmic Rays in Interplanatory Magnetic Fields* (Reidel, Dordrecht, 1985), p. 313.